Traversable Wormholes for Time Travel: Morris-Thorne Wormhole

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Wormholes are phenomena of general relativity that are of particular interest to science fiction as they may allow time travel. Here, I describe a simple solution to Einstein's equations that represents a wormhole – the Morris-Thorne wormhole [1] – that connects two regions of flat spacetime while allowing human travelers to pass through in principle. The stress-energy tensor that's required to maintain such wormholes is derived, and the wormhole's geometry is visualized through an embedding diagram. The possibility of using this wormhole for backwards time travel and the feasibility of the stress-energy tensor are also discussed.

1. INTRODUCTION

Among the numerous fantasies of science fiction, time travel is one of the most intriguing. Fortunately, there are solutions to Einstein's field equation that connect two regions of asymptotically flat spacetime through a "throat". The two spacetimes at the two ends of the "throat" could be different galaxies, or far regions within the same galaxy. Supposedly, if a traveler enters the "throat" at one end and exits through another, she would find herself in a distant region of our universe or another universe. The possibility of using this mechanism for backward time travel is discussed in Section 4A.

One example of such a wormhole is the Schwarzschild wormhole, which is described by the Schwarzschild metric but connects two regions of flat spacetime. However, serious difficulties prevent them from practical use. They have horizons similar to Schwarzschild blackholes, which kill any traveler attempting to traverse them by enormous tidal forces. Additionally, they are not static, and their throat shrinks to zero circumference so rapidly that even light cannot pass through. [1]

So what does it mean for a wormhole to be traversable? Here, several conditions are listed, and they serve as broad design specifications for the wormhole. We assume that we are working under the framework of general relativity, and thus the solutions obey the Einstein field equations.

1. The wormhole connects two asymptotically flat regions of spacetime through a throat. A qualitative diagram is shown in Figure 1.

2. There are no horizons, as we want the wormhole to be traversable from both sides.

3. The tidal gravitational force experienced by travelers needs to be reasonable for humans passing through.

4. The wormhole needs to be traversable in a reasonable amount of time, as measured both by the traveler as well as a stationary observer.

5. The stress-energy tensor that produces the wormhole needs to be physically reasonable. This will be further discussed in Section 4B.

6. The wormhole needs to be stable under perturbations.

The Morris-Thorne wormhole is a class of wormholes that satisfy much of the above requirements (except 5 which is especially troublesome). In Section 2, I will derive the required stress-energy tensor for maintaining the wormhole. In Section 3, the wormhole geometry will be visualized by plotting the equatorial plane embedded in 3D space. In Section 4, the implications of the stress-energy tensor and how backward time travel can be achieved will be explored. Most calculations in this paper are done with symbolic programs and can be found in this notebook.

2. MATHEMATICAL DETAILS

A. The Metric and the Stress-Energy Tensor

We begin with the static, spherically symmetric metric:

$$ds^2 = -e^{2\Phi}dt^2 + \frac{1}{1-\frac{b}{r}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad (1)$$

where $\Phi = \Phi(r)$ and $b = b(r)$ are functions of radius only. These will be used later to address constraints on traversability and stability. Here, it is apparent that as $r \to \infty$, the metric approaches the Minkowski metric, which indicates that two regions of flat spacetime is connected. As $\frac{b}{r} \to 1$, we run into a coordinate singularity, which is the throat of the wormhole. From here, traversing through the wormhole corresponds to a traveler sitting at $r = \infty$ and then moving towards the throat of the wormhole at $r = b_0$, and then coming out of the other side at $r = \infty$.

From this metric, we can calculate the Christoffel symbols and the Riemann curvature tensor using standard formulas given by:

$$\Gamma^{\alpha}_{\beta\gamma} = \frac{1}{2}g^{\alpha\sigma}\left(g_{\sigma\gamma,\beta} + g_{\sigma\beta,\gamma} - g_{\beta\gamma,\sigma}\right) \quad (2)$$

$$R^{\alpha}_{\beta\gamma\delta} = \Gamma^{\alpha}_{\beta\delta,\gamma} - \Gamma^{\alpha}_{\beta\gamma,\delta} + \Gamma^{\alpha}_{\sigma\delta}\Gamma^{\sigma}_{\beta\gamma} - \Gamma^{\alpha}_{\sigma\gamma}\Gamma^{\sigma}_{\beta\delta} \tag{3}$$

The non-zero components of the Riemann curvature tensor are:

$$\begin{aligned}
R^{t}_{rtr} &= -R^{t}_{rrt} = -\Phi'' - \Phi'^2 + \frac{(b'r-b)\Phi'}{2r(r-b)} \\
R^{r}_{ttr} &= -R^{r}_{trt} = (1-\frac{b}{r})e^{2\Phi}(-\Phi'' - \Phi'^2 + \frac{(b'r-b)\Phi'}{2r(r-b)}) \\
R^{t}_{\theta t\theta} &= -R^{t}_{\theta\theta t} = (b-r)\Phi' \\
R^{t}_{\phi t\phi} &= -R^{t}_{\phi\phi t} = (b-r)\Phi'\sin^2\theta \\
R^{r}_{\theta r\theta} &= -R^{r}_{\theta\theta r} = \frac{rb'-b}{2r} \\
R^{r}_{\phi r\phi} &= -R^{r}_{\phi\phi r} = \frac{rb'-b}{2r}\sin^2\theta \\
R^{\theta}_{\phi\theta\phi} &= -R^{\theta}_{\phi\phi\theta} = \frac{b\sin^2\theta}{r} \\
R^{\phi}_{\theta\phi\theta} &= -R^{\phi}_{\theta\theta\phi} = \frac{b}{r} \\
R^{\theta}_{tt\theta} &= -R^{\theta}_{t\theta t} = R^{\phi}_{t\phi t} = -R^{\phi}_{tt\phi} = \frac{(b-r)\Phi' e^{2\Phi}}{r^2} \\
R^{\theta}_{r\theta r} &= -R^{\theta}_{rr\theta} = R^{\phi}_{r\phi r} = -R^{\phi}_{rr\phi} = \frac{rb'-b}{2r^2(r-b)}
\end{aligned} \tag{4}$$

Here, the $'$ denotes a derivative with respect to r. We then switch to orthonormal coordinates to see what a stationary observer sees as the stress-energy tensor. The orthonormal coordinate basis vectors are given by:

$$\begin{aligned}
e_t = e^{-\Phi}\partial_t, \quad e_r = \sqrt{1-\frac{b}{r}}\partial_r, \\
e_\theta = \frac{1}{r}\partial_\theta, \quad e_\phi = \frac{1}{r\sin\theta}\partial_\phi
\end{aligned} \tag{5}$$

We transform the Riemann curvature tensor to orthonormal coordinates using the formula:

$$R^{\hat{\alpha}}_{\hat{\beta}\hat{\gamma}\hat{\delta}} = e^{\hat{\alpha}}_{\mu} e^{\nu}_{\hat{\beta}} e^{\rho}_{\hat{\gamma}} e^{\sigma}_{\hat{\delta}} R^{\mu}_{\nu\rho\sigma} \tag{6}$$

where $e^{\hat{\alpha}}_{\mu}$ is the inverse of the orthonormal basis vectors. They have values as follows:

$$\begin{aligned}
e^{t}_{\hat{t}} &= (e^{\hat{t}}_{t})^{-1} = e^{-\Phi} \\
e^{r}_{\hat{r}} &= (e^{\hat{r}}_{r})^{-1} = \sqrt{1-\frac{b}{r}} \\
e^{\theta}_{\hat{\theta}} &= (e^{\hat{\theta}}_{\theta})^{-1} = \frac{1}{r} \\
e^{\phi}_{\hat{\phi}} &= (e^{\hat{\phi}}_{\phi})^{-1} = \frac{1}{r\sin\theta}
\end{aligned} \tag{7}$$

Now, we calculate the components of the Riemann curvature tensor in orthonormal coordinates. The non-zero components are:

$$\begin{aligned}
R^{\hat{t}}_{\hat{r}\hat{t}\hat{r}} &= -R^{\hat{t}}_{\hat{r}\hat{r}\hat{t}} = R^{\hat{r}}_{\hat{t}\hat{t}\hat{r}} = -R^{\hat{r}}_{\hat{t}\hat{r}\hat{t}} \\
&= (1-\frac{b}{r})(-\Phi'' - \Phi'^2 + \frac{(b'r-b)\Phi'}{2r(r-b)}) \\
R^{\hat{t}}_{\hat{\theta}\hat{t}\hat{\theta}} &= -R^{\hat{t}}_{\hat{\theta}\hat{\theta}\hat{t}} = R^{\hat{\theta}}_{\hat{t}\hat{t}\hat{\theta}} = -R^{\hat{\theta}}_{\hat{t}\hat{\theta}\hat{t}} \\
= R^{\hat{t}}_{\hat{\phi}\hat{t}\hat{\phi}} &= -R^{\hat{t}}_{\hat{\phi}\hat{\phi}\hat{t}} = R^{\hat{\phi}}_{\hat{t}\hat{t}\hat{\phi}} = -R^{\hat{\phi}}_{\hat{t}\hat{\phi}\hat{t}} = \frac{(b-r)\Phi'}{r^2} \\
R^{\hat{r}}_{\hat{\theta}\hat{r}\hat{\theta}} &= -R^{\hat{r}}_{\hat{\theta}\hat{\theta}\hat{r}} = R^{\hat{\theta}}_{\hat{r}\hat{r}\hat{\theta}} = -R^{\hat{\theta}}_{\hat{r}\hat{\theta}\hat{r}} \\
= R^{\hat{r}}_{\hat{\phi}\hat{r}\hat{\phi}} &= -R^{\hat{r}}_{\hat{\phi}\hat{\phi}\hat{r}} = R^{\hat{\phi}}_{\hat{r}\hat{r}\hat{\phi}} = -R^{\hat{\phi}}_{\hat{r}\hat{\phi}\hat{r}} = \frac{(rb'-r)}{2r^3} \\
R^{\hat{\theta}}_{\hat{\phi}\hat{\theta}\hat{\phi}} &= -R^{\hat{\theta}}_{\hat{\phi}\hat{\phi}\hat{\theta}} = R^{\hat{\phi}}_{\hat{\theta}\hat{\phi}\hat{\theta}} = -R^{\hat{\phi}}_{\hat{\theta}\hat{\theta}\hat{\phi}} = \frac{b}{r^3}
\end{aligned} \tag{8}$$

We can then contract the Riemann tensor to obtain the Ricci tensor $R_{\hat{\mu}\hat{\nu}}$ and scalar curvature R, and subsequently the Einstein tensor $G_{\hat{\mu}\hat{\nu}}$ using the formulas:

$$R_{\hat{\mu}\hat{\nu}} = R^{\hat{\alpha}}_{\hat{\mu}\hat{\alpha}\hat{\nu}} \tag{9}$$

$$R = g^{\hat{\mu}\hat{\nu}} R_{\hat{\mu}\hat{\nu}} \tag{10}$$

$$G_{\hat{\mu}\hat{\nu}} = R_{\hat{\mu}\hat{\nu}} - \frac{1}{2} g_{\hat{\mu}\hat{\nu}} R \tag{11}$$

The resulting Einstein tensor is:

$$\begin{aligned}
G_{\hat{t}\hat{t}} &= \frac{b'}{r^2} \\
G_{\hat{r}\hat{r}} &= -\frac{b}{r^3} + \frac{2(1-\frac{b}{r})\Phi'}{r} \\
G_{\hat{\theta}\hat{\theta}} &= G_{\hat{\phi}\hat{\phi}} = \\
&(1-\frac{b}{r})(\Phi'' + \Phi'^2 - \frac{(b'r-b)\Phi'}{2r(r-b)} + \frac{\Phi'}{r} - \frac{b'r-b}{2r^2(r-b)})
\end{aligned} \tag{12}$$

Since the Einstein field equations are $G_{\hat{\alpha}\hat{\beta}} = 8\pi T_{\hat{\alpha}\hat{\beta}}$, the stress-energy tensor $T_{\hat{\alpha}\hat{\beta}}$ is obtained by a scalar multiple of $G_{\hat{\alpha}\hat{\beta}}$.

B. A Simple Specific Solution

The functions $\Phi(r)$ and $b(r)$ can be chosen to satisfy the requirements of no horizons (2), low tidal force (3), and traversable in a reasonable time (4) [1]. Here, for simplicity, we choose $\Phi(r) = 0$ which corresponds to no redshift from the wormhole, and $b(r) = \frac{b_0^2}{r}$, which specifies the shape of the wormhole. b_0 is a constant that specifies the radius of the throat of the wormhole. The metric and the stress-energy tensor now simplifies to:

$$ds^2 = -dt^2 + \frac{1}{1-\frac{b_0^2}{r^2}}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{13}$$

$$-T_{\hat{t}\hat{t}} = -T_{\hat{r}\hat{r}} = T_{\hat{\theta}\hat{\theta}} = T_{\hat{\phi}\hat{\phi}} = \frac{b_0^2}{8\pi r^4} \tag{14}$$

For this simplified solution, the stress-energy tensor has negative energy density. This violates requirement 5, but is an unavoidable problem (see Section 4B). Furthermore, it can be shown that the tidal forces felt by a traveler can be made arbitrarily small depending on the velocity at which she travels through the wormhole.

3. VISUALIZATION

We now visualize the simplified metric in equation 13 to gain an intuitive understanding of the wormhole's geometry. For this section we mostly follow the steps shown in Sec 23.8 of [2].

Since the metric is time independent as well as spherically symmetric, we set $\theta = \pi/2$ and $t = 0$ to obtain a 2D slice of the wormhole that corresponds to the equatorial plane and embed it in 3D space. Since $d\theta = 0$ and $dt = 0$, the geometry on this slice is given by the line element:

$$ds^2 = \frac{1}{1 - \frac{b_0^2}{r^2}} dr^2 + r^2 d\phi^2 \tag{15}$$

We will use cylindrical coordinates (r, ϕ, z) for the embedded space as the resulting geometry is axially symmetric. Here, the r and the ϕ coordinates are the same as the original metric, while the z coordinate will be given by an embedding function $z(r)$. The line element of the slice in cylindrical coordinates is:

$$ds^2 = dz^2 + dr^2 + r^2 d\phi^2 \tag{16}$$

which can also be written as:

$$ds^2 = (1 + (\frac{dz}{dr})^2) dr^2 + r^2 d\phi^2 \tag{17}$$

This should match the line element of the slice in equation 15, giving us the differential equation describing the embedding function $z(r)$:

$$\begin{aligned} 1 + (\frac{dz}{dr})^2 &= \frac{1}{1 - \frac{b_0^2}{r^2}} \\ \implies \frac{dz}{dr} &= \pm \frac{1}{\sqrt{(\frac{r}{b_0})^2 - 1}} \end{aligned} \tag{18}$$

After integration, we obtain:

$$z(r) = \pm b_0 \ln(|\frac{r}{b_0} + \sqrt{(\frac{r}{b_0})^2 - 1}|) + C \tag{19}$$

Ignoring the $+C$ and setting $b_0 = 1$, Figure 1 shows the embedding of the wormhole in 3D space. The top and bottom part extends outwards radially and approach flat spacetime. The throat of the wormhole is at $z = 0$, which is the region of minimum circumference.

4. DISCUSSION

A. Possibility of Backward Time Travel

The idea of converting a wormhole into a time machine is proposed in [3]. Metaphorically, it works similarly to the twin paradox from special relativity. One of the twins stay on Earth while the other boards a spaceship and travels near light speed to a distant planet and then back to Earth. The person who stayed on Earth is 100 years old, and the person who came back from space is 50 years old. If we had given the twins two ends of a wormhole and have them carry their wormhole mouth, by the time the person who went to space returns, one end of the wormhole would be 100 years old and the other end 50 years old. Going through the wormhole from one end to another, we can either travel back **or** travel forward in time for 50 years. An advanced civilization may be able to "pull" the wormhole mouth to near light speed through gravity, thus building a time machine.

Being able to travel backwards in time results in closed timelike curves and can violate causality. One can imagine it would be possible to travel backwards in time and kill a younger self before entering the wormhole. Scenarios like this have spurred speculation that a wormhole cannot be constructed at all.

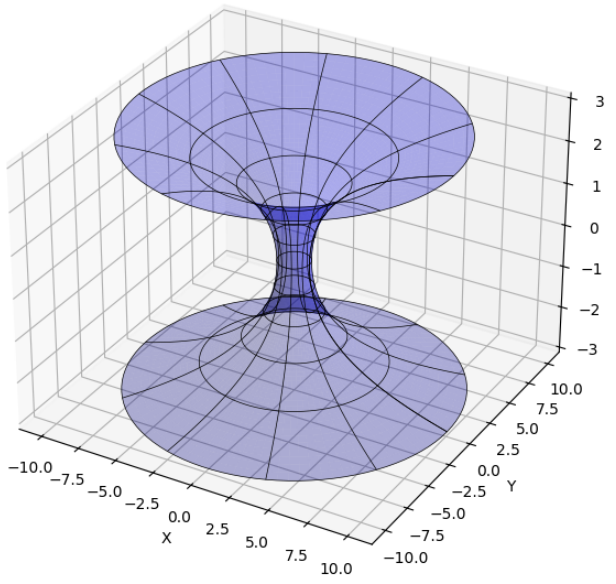


FIG. 1. Embedding of the equatorial slice of the wormhole in 3D space. Code for plotting is here.

B. Violation of the Weak-Energy Condition

The stress-energy tensor obtained from 14 has negative energy density. Although this is not a generic property of wormholes, any traversable wormhole must have some "exotic matter" seen by some observer [1]. Since wormholes are defined as connecting two asymptotically flat spacetimes through a throat, bundles of light rays that enter the wormhole at one end will have their cross-sectional area decreased, and then increased as they exit from the other end. This increase of cross-sectional area can only be done with energy that repulses, hence the necessity of negative energy density seen by some observer.

It is known that quantum field theory permits violations of the weak-energy condition, which allows negative energy densities to exist. However, wormholes violate a stronger version of the condition, the averaged weak-energy condition, which states that any trajectory-averaged energy density must be positive [3]. It is currently unclear whether quantum field theory permits a violation of this stronger condition, so hopes of constructing a traversable wormhole and thus a time machine are not lost yet.

[1] M. S. Morris and K. S. Thorne, American Journal of Physics **56**, 395 (1988).
[2] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (W. H. Freeman, San Francisco, 1973).
[3] M. S. Morris, K. S. Thorne, and U. Yurtsever, Phys. Rev. Lett. **61**, 1446 (1988).